Analysis of Conjoint Data: Part III: Latent Class Analysis

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Preference Heterogeneity

- **Reminder:**

The probability that an individual will choose i from the choice set C composed of n stimuli:

$$p(i|C) = \frac{exp(U_i)}{\sum_{j=1}^{m} exp(U_j)} = \frac{exp(x_i \boldsymbol{\beta})}{\sum_{j=1}^{m} exp(x_j \boldsymbol{\beta})}$$

Homogeneous across consumers!

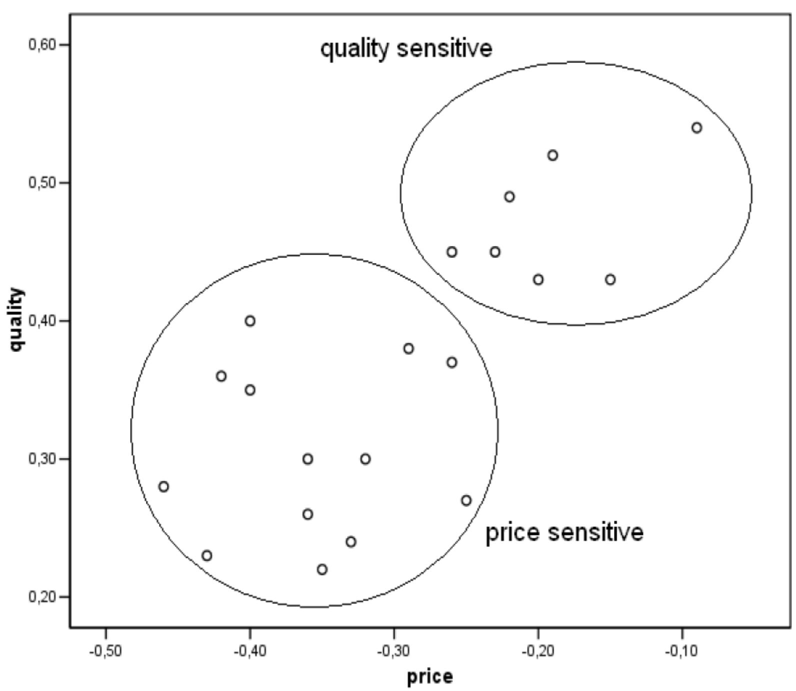
Latent Class Analysis:
Heterogeneous across segments of consumers

Latent Class Model

- **Multinomial Logit Model** (MNL):

We now assume that this probability will depend on the segment s this individual belongs to:

$$p_{s}(i|C) = \frac{exp(U_{is})}{\sum_{j=1}^{m} exp(U_{js})} = \frac{exp(x_i \beta_s)}{\sum_{j=1}^{m} exp(x_j \beta_s)}$$



Example of a market with 2 segments:

- Low-price segment
  ➔Highly negative part-worth for price
- High-quality segment:
  ➔Highly positive part-worth for quality

Latent Class Model

- In contrast to clustering (e.g. k-means), we are not 100% sure what segment an individual belongs to

$p_{(s=1)}$ = Probability that individual is in segment 1

$p_{(s=2)}$ = Probability that individual is in segment 2

$\dots$

$p_{(s=S)}$ = Probability that individual is in segment S

Latent Class Model

- Therefore, the probability that an individual will choose *i* in choice set *C*

$$p(i|C) =$$

Proba. of choosing i given part-worths of segment 1

$$p_{(s=1)} \quad \boxed{\times \; p_1(i|C) \; +}$$

$$p_{(s=2)} \quad \times \; p_2(i|C) \; +$$

$$\ldots \quad \ldots$$

See before

$$p_{(s=S)} \quad \times \; p_S(i|C)$$

With $p_s(i|C) = \frac{exp(x_i\beta_s)}{\sum_{j=1}^{m} exp(x_j\beta_s)}$

Intuition:

Take the weighted average probability of choosing *i* across all segments, with the weights given by the probability of the individual to belong to that segment

Example

- Segments:
  - Segment 1: Low-price segment
  - Segment 2: High-quality segment

- Respondents:
  - Respondent 1: $p_{(s=1)} = 0.80$, $p_{(s=2)} = 0.20$
  - Respondent 2: $p_{(s=1)} = 0.60$, $p_{(s=2)} = 0.40$
  - Respondent 3: $p_{(s=1)} = 0.50$, $p_{(s=2)} = 0.50$

- Interpretation:
  - Respondents 1 and 2 probably belong to low-price segment but evidence is stronger for respondent 1 than for respondent 2
  - For respondent 3, there is as much evidence she belongs to any segment

Example (continued)

- Suppose a choice set C with 3 products:

Segment s	Proba to belong to
1	0.7
2	$\boxed{0.3}$

p_2

Segment s	Probability of choosing product i		
	$i = 1$	$i = 2$	$i = 3$
1	0.6	0.3	0.1
2	0.3	0.2	$\boxed{0.5}$

$p_2(3|C)$

$\Rightarrow$ Probability of choosing alternative 3 from the set C

$$= p(3|C)$$

$$= p_{(s=1)} \times p_1(3|C) + p_{(s=2)} \times p_2(3|C)$$

$$= 0.7 \times 0.1 + 0.3 \times 0.5$$

Segmentation Principles

- Which refinement level to consider? *Segmentation level*
  - Macro: country segments (e.g. Europe vs. Asia)
  - Micro: consumer segments (e.g. young vs. old; price-sensitive vs. quality-seeking)

- Which distance(s) to consider? *Segment basis*
  - <u>*General basis*</u>: independent of the domain, (i) <u>observable</u>: geographic regions (Middle East, Oceania), socio-demographic variables (population size, age, education, language), … or (ii) <u>unobservable</u>: cultural dimensions (Hofstede, Schwartz values), life styles (VALS value-attitude-lifestyle).
  - <u>*Domain-specific basis*</u>: type of usage (heavy vs. light), financial product ownership, brand loyalty

- Which method to use to find the segment? *Segmentation method*

Criteria for a Good Segmentation

- Six factors determining the effectiveness of market segmentation:
  1. *Identifiability*: easily measured segmentation bases
  2. *Substantiality*: segments should be large enough to be profitable
  3. *Accessibility*: effective promotional/distributional tools to reach segment(s)
  4. *Stability*: composition of segments should not change rapidly
  5. *Responsiveness*: homogeneous, unique response within segment
  6. *Actionability*: segments and firm's goals/competencies should match

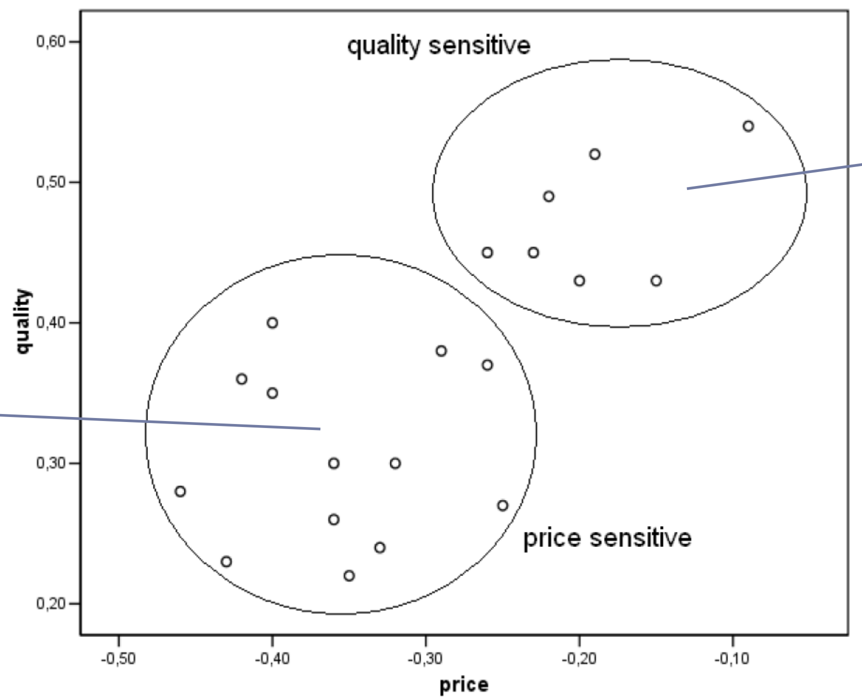
- CBC data
  1. Add profiling variables (next slide)
  2. Check segment sizes, use IC criteria (see later slides)
  3. Add channel check; add price (or promotion) attribute
  4. Stable preferences for attributes
  5. Select attributes' levels according to a segment's preferences
  6. Focus on segments with preferred levels in line with firm's strategy

Segmentation Methods

- Clustering methods
  - E.g. k-means
  - Can be used with CBC data but in two steps:
    - Step 1: estimate the part-worths per customer
    - Step 2: apply k-means on these part-worths (rows: customers, columns: part-worths)
- Latent-Class analysis
  - Does it in one step!

Profiling Segments

- Connecting preferences to observed socio-demographics
- We can predict preferences for any (new) customer, even those who have not filled in the CBC as soon as we know her socio-demographics



Older customers
Male
Higher income

Younger customers
Female
Lower income

- Increase the **identifiability** of the segments!

Prior and Posterior Probabilities

Two types of segment membership probabilities:

- Prior probability:
  - Probability of segment membership before observing individual data
  - Prior probability = proportion of individuals in a segment (segment size)

  Not specific for individuals

- Posterior probability:
  - Probability of segment membership after observing individual data
  - Respondent's choices from choice sets represent useful segment info

  Individual specific

Prior and Posterior Probabilities

Prior probability to belong to segment *s* → Observe choice of each customer → Posterior probability for a customer to belong to segment *s*

Example: 2 segments, 2 alternative stimuli, one choice is observed

Two segments:

Low-price segment (50% of the customers)
High-quality segment (50% of the customers)

Two scenarios:

a) Customer chooses a low-price stimulus
b) Customer chooses a high-quality stimulus

Membership probability for customer i depending on scenario:

If scenario a), proba. to belong to the low-price segment ↑
If scenario b), proba. to belong to the low-price segment ↓

Example

- Assume two equal-sized segments:
  - Pr(low-price segment) = 0.50,
  - Pr(high-quality segment) = 0.50

Prior probabilities

- Suppose one choice set with 2 products
  - Product A: low-price product
  - Product B: high-quality product

- We observe:
  - 80% of customers belonging to the low-price segment choose A
  - 30% of customers belonging to the high-quality segment choose A

Likelihood(A|low-price) = 0.80 ➔ Likelihood (B|low-price) = 0.20

Likelihood(A|high-quality) = 0.30 ➔ Likelihood (B|high-quality) = 0.70

➔ these are called likelihood of choice per segment: Likelihood(product *i*|segment *s*)

- We want posterior segment probabilities: Pr(segment s|product *i*)

Example

	A	B	
price-sens (50)	80%	20%	**100%**
quality-sens (50)	30%	70%	**100%**

	A	B	
price-sens	$50 \times 0.80 = 40$	$50 \times 0.20 = 10$	**50**
quality-sens	$50 \times 0.30 = 15$	$50 \times 0.70 = 35$	**50**
	55	**45**	**100**

Example

	A	B	
price-sens	$50 \times 0.80 = 40$	$50 \times 0.20 = 10$	**50**
quality-sens	$50 \times 0.30 = 15$	$50 \times 0.70 = 35$	**50**
	55	**45**	**100**

	A	B
price-sens	$100\% \times \frac{40}{55} = 73\%$	$100\% \times \frac{10}{45} = 22\%$
quality-sens	$100\% \times \frac{15}{55} = 27\%$	$100\% \times \frac{35}{45} = 78\%$
	100%	**100%**

Example

- Hence, after observing respondent's choice:
  - If alternative A is chosen, posteriors become
    - Pr(low-price segment) is updated from 50% to 73%
    - Pr(high-quality segment) is updated from 50% to 27%
  - If alternative B is chosen, posteriors become
    - Pr(low-price segment) is updated from 50% to 22%
    - Pr(high-quality segment) is updated from 50% to 78%

Example: Summary

	A	B	
price-sens	$50 \times 0.80 = 40$	$50 \times 0.20 = 10$	**50**
quality-sens	$50 \times 0.30 = 15$	$50 \times 0.70 = 35$	**50**
	55	**45**	**100**

	A	B
price-sens	$\frac{0.50 \times 0.80}{0.50 \times 0.80 + 0.50 \times 0.30} = 0.73$	$\frac{0.50 \times 0.20}{0.50 \times 0.20 + 0.50 \times 0.70} = 0.22$
quality-sens	$\frac{0.50 \times 0.30}{0.50 \times 0.80 + 0.50 \times 0.30} = 0.27$	$\frac{0.50 \times 0.70}{0.50 \times 0.20 + 0.50 \times 0.70} = 0.78$
	1.00	**1.00**

Formalization

Fraction of customers in segment s

$$\boxed{\text{Posterior}(s)} = \frac{\boxed{\text{prior}(s)} \times \boxed{\text{likelihood}(i\,|\,s)}}{\Sigma_s\,\text{prior}(s) \times \text{likelihood}(i\,|\,s)}$$

Proba for a customer to belong to segment s after having chosen product i

Likelihood (i.e. probability) that a customer chooses product *i* given the preferences (i.e. part-worths) of segment s

$$\boxed{\Pr(i\,|\,s)} = \frac{\boxed{p_{(s=s)}} \times \boxed{\text{Likelihood}(i\,|\,s)}}{\sum_s p_{(s=s)} \times \text{Likelihood}(i\,|\,s)}$$

With Likelihood(i|s)=$p_s(i|C) = \frac{exp(U_{is})}{\sum_{j=1}^{m} exp(U_{js})} = \frac{exp(x_i\beta_s)}{\sum_{j=1}^{m} exp(x_j\beta_s)}$

How Many Segments?

- Adding segment increases model fit by construction, but is increase sufficient to justify increased model complexity?

$\Rightarrow$ Tradeoff between model fit & complexity (# parameters)

Fit is good >< Complexity is bad

- Popular and simple approach: try different numbers of segments, and **minimize** some "information criterion" balancing fit and complexity

$$\text{Information Criterion (IC)} = -2 \times LL + P \times npar$$

- With
  - LL = (natural) log-likelihood (given by sawtooth)
  - P = penalty (depends on the criterion chosen)
  - npar = number of estimated parameters

How Many Segments?

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- Popular and simple approach: try different numbers of segments, and **minimize** some "information criterion" balancing fit and complexity

Information Criterion (IC) = $-2 \times$ LL + P $\times$ npar

- Most common ones:

Akaike (AIC): largest number of segments (too many?)
Bayesian (BIC): most popular in literature
Consistent Akaike (CAIC): smallest number of segments

How Many Segments?

Information criterion (IC): $-2 \times LL + P \times npar$

		Penalty P
Akaike	AIC	2
Bayes	BIC	ln(nb. obs)
Consistent Akaike	CAIC	ln(nb. obs) + 1

How Many Segments?

Information criterion (IC): $-2 \times LL + P \times npar$

		Penalty P	Nb obs. = 10	Nb obs.= 100	Nb obs. = 1000
Akaike	AIC	2	2	2	2
Bayes	BIC	ln(nb. obs)	2.30	4.61	6.91
Consistent Akaike	CAIC	ln(nb. obs) + 1	3.30	5.61	7.91

BIC and CAIC give higher penalties ➔ they favor less segments

Golf Ball Data – Estimation Summary

	A	B	C	D	E	F	G	H	I	J
1	**Latent Class Estimation**									
2										
3	Minimum number of groups	1								
4	Maximum number of groups	8								
5	Number of replications	5								
6	Maximum number of iterations	100								
7	Convergence limit for log likelihood	0.01000								
8	Standard errors reported									
9	All pairs of solutions will be tabulated									
10	Random number seed	2562								
11										
12	Null log-likelihood	-5198.60385								
13										
14	**Summary of best replications**									
15	Groups	Replication	Log-likelihood	Pct Cert	AIC	CAIC	BIC	ABIC	Chi-Square	Relative Chi-Square
16	1	3	-4600.39430	11.50712	9218.78860	9283.85420	9274.85420	9246.25651	1196.41911	132.93546
17	2	1	-3994.88613	23.15463	8027.77226	8165.13297	8146.13297	8085.76008	2407.43545	126.70713
18	3	3	-3872.78105	25.50344	7803.56210	8013.21792	7984.21792	7892.06983	2651.64561	91.43606
19	4	4	-3830.12894	26.32389	7738.25789	8020.20882	7981.20882	7857.28552	2736.94982	70.17820
20	5	4	-3798.74222	26.92765	7695.48444	8049.73049	8000.73049	7845.03198	2799.72326	57.13721
21	6	5	-3775.86204	27.36777	7669.72408	8096.26524	8037.26524	7849.79152	2845.48363	48.22854
22	7	3	-3760.20059	27.66903	7658.40119	8157.23746	8088.23746	7868.98853	2876.80652	41.69285
23	8	2	-3747.06254	27.92175	7652.12508	8223.25646	8144.25646	7893.23233	2903.08263	36.74788

Models estimated for 1 to 8 segments

Based on CAIC, we select 3 segments

Golf Ball Data – Segment Sizes

	A	B	C	D
31	Adjusted Bayesian Info Criterion	7892.11997		
32	Chi-Square	2651.59547		
33	Relative Chi-Square	91.43433		
34				
35	**Segment Sizes**	24.2%	14.9%	60.9%
36				
37	**Part Worth Utilities**			
38	High-Flyer Pro, by Smith and Forester	1.10892	4.41737	0.40825

Summary | 1 Groups | 2 Groups | **3 Groups** | 4 Groups

- **Ensure that segments are large enough (substantiality)**
  - Rule of thumb: at least 10%
- **Segment sizes represent prior probabilities**

Golf Ball Data – Partworths Per Segment

	A	B	C	D
37	**Part Worth Utilities**			
38	High-Flyer Pro, by Smith and Forester	1.10892	4.41737	0.40825
39	Magnum Force, by Durango	0.51904	4.36426	0.31703
40	Eclipse+, by Golfers, Inc.	-0.74320	2.18729	-0.28384
41	Long Shot, by Performance Plus	-0.88477	-10.96891	-0.44144
42				
43	Drives 5 yards farther than the average ball	-0.91240	-1.59377	-0.36313
44	Drives 10 yards farther than the average ball	0.35230	0.29488	0.08693
45	Drives 15 yards farther than the average ball	0.56010	1.29890	0.27621
46				
47	$4.99 for package of 3 balls	1.28277	4.14243	0.49397
48	$6.99 for package of 3 balls	0.11582	3.02674	0.21384
49	$8.99 for package of 3 balls	-0.21453	2.81643	-0.07400
50	$10.99 for package of 3 balls	-1.18406	-9.98560	-0.63381
51				
52	NONE	0.78648	9.96769	-2.09669
53				

- For every segment study partworths and t-ratios
- Compare partworths within a segment only

Golf Ball Data – Partworths Across Segments

	A	B	C	D
88	**Part Worth Utilities Rescaled for Comparability**			
89	High-Flyer Pro, by Smith and Forester	56.07215	40.89273	46.80347
90	Magnum Force, by Durango	26.24500	40.40112	36.34517
91	Eclipse+, by Golfers, Inc.	-37.57945	20.24830	-32.54056
92	Long Shot, by Performance Plus	-44.73770	-101.54215	-50.60808
93				
94	Drives 5 yards farther than the average ball	-46.13519	-14.75399	-41.63101
95	Drives 10 yards farther than the average ball	17.81411	2.72975	9.96559
96	Drives 15 yards farther than the average ball	28.32109	12.02424	31.66542
97				
98	$4.99 for package of 3 balls	64.86263	38.34756	56.63014
99	$6.99 for package of 3 balls	5.85634	28.01931	24.51580
100	$8.99 for package of 3 balls	-10.84772	26.07246	-8.48406
101	$10.99 for package of 3 balls	-59.87124	-92.43933	-72.66188
102				
103	NONE	39.76780	92.27353	-240.37258

- **Partworths are rescaled to facilitate comparison across segments**
  [Rescaling such that average attribute range is 100 within every segment]

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- **Eclipse+ brand relatively more preferred by segment 2**
  [Caution: t-statistic insignificant (.03); for other segments negative significant]

Golf Ball Data – Attribute Importances

	A	B	C	D
105	**Attribute Importances**			
106	Brand:	33.60328	47.47829	32.47052
107	Perfomance:	24.81876	8.92608	24.43214
108	Price:	41.57796	43.59563	43.09734
109				
110	The average maximum membership probability is 0.96302.			

Segment 2 …

… seems to care little for Performance compared to segment 1 & 3

… attaches more importance to Brand compared to segment 1 & 3